

forges ahead into elastic-plastic natural behavior and combination of BE and FE. These chapters are most interesting and should be read by those interested in plasticity.

Chapter 9 deals with finite deflections of plates. Until previously most nonlinear problems pertaining to plates used simplified nonlinear differential equations. Examples of this are Berger's equation, von Karman's nonlinear differential equation for finite deflection of plates, shells, and others. Now, BE method becomes a very effective tool in dealing with the analysis for nonlinear as well as linear problems. Initiating the subject, the authors review the well-known Kirchhoff-Love equations referred to in Cartesian coordinates and von Karman equations. The integral equation for boundary elements are now the starting point. After casting the von Karman equations in matrix form, an iterative process is employed in integrating them numerically, Berger's equation neglects, without justification, the second invariant of the membrane stress in the potential energy expression for laterally loaded, homogeneous, isotropic thin plates. Nevertheless, this equation has been used extensively by investigators in dynamic and static problems for shallow shells, sandwich plates, and others. The Berger equations are then formulated in terms of integral equations and is thus available for numerical computation. The resulting numerical scheme and system of simultaneous equations are similar to those applied in linear bending. Proceeding from plates, we journey to nonlinear shell and sandwich plate/shell problems. Based upon the foregoing analysis, this opens up an avenue in deriving integral formulations in studies other than Berger and von Karman field equations. This chapter comes to the point and should be read by those interested in plate and shell analysis.

The last chapter points out the advantage of using the Trefftz method cast in boundary integral equations. The prime purpose of his method is the investigation of approximate solutions obtained from the appropriate class of functions which satisfy exactly the differential equation but may not exactly satisfy the prescribed boundary conditions. The author develops the necessary recipes for the BE method and continues ahead to Green's formulas. Illustrations are provided using the latter in both the continuous and discontinuous fields. The chapter continues with the derivation of Laplace and reduced wave equation in both two and three dimensions on the T-complete system. The concluding section briefly mentions the Hilbert space formulation. This is a very good chapter and must be read carefully due to its mathematical content.

In summary, this is an excellent book on BE methods. Although not an elementary text, it does furnish a good understanding of the BE method. The reviewer would have preferred seeing chapters on BE method applied to a greater depth in acoustics and dynamics. Perhaps, this will be covered in later volumes. The reviewer recommends this book to those interested in the BE method.

**Finite Elements: Mathematical Aspects, Volume IV.** by J. T. Oden and G. F. Carey, Prentice-Hall, Englewood Cliffs, NJ, 1983, 195 pages. Price: \$37.95

This book is not meant for the "faint-hearted." The authors have written a book which proves in a methodical fashion the mathematical foundations of the finite element (FE) method. It is one of six volumes in this series. The book is essentially a mathematical text with very few illustrative examples. Two of the other volumes deal with solid mechanics and fluid mechanics. As stated by the authors, "The treatment is by no means comprehensive; rather the aim is to present only the basic mathematical properties of FE needed to understand how these methods work and how their convergence and

stability properties can be established for linear elliptical properties. The subject presented here summarizes some of the important developments in the mathematical theory of FE completed since the 1970s." The authors accomplish this task but at times require extensive concentration and patience.

The book consists of five chapters.

Chapter 1 introduces the subject and shows how the modern theory of elliptic boundary-value problems (BVP) and the related FE approximations are based on the classical concept of variations. The authors begin with Banach spaces and continue to Sobolev spaces, Sobolev embedding theorem plus variational and approximation of variational BVP. Chapter 2 introduces reader to the finite element interpolation theory. The FE methods shows properties similar to the popular curve-fitting techniques and generally represents the function as a polynomial in the same sense as the Lagrangian and Hermitian interpolation methods. The interpolation of the functions is derived in Sobolev space. Some general properties of FE are considered as to assembly, global degrees of freedom,  $P$ -unisolvant elements, i.e., a unique function containing polynomials of degree  $P$ . The latter contains the Bogner-Fox-Schmidt rectangular element and the Hermitian elements. Next, the affine family is considered where two finite elements of the same type are equivalent and can be mapped one on to the other. The chapter concludes with the interpolation theory in Sobolev spaces. This is good chapter that requires intensive concentration.

Chapter 3 focuses on the FE approximation of elliptical BVP. This centers around the idea of sharpening up the rate of convergence of standard FE methods as the mesh parameter tends to zero. Beginning with standard error estimates, this leads to the question of how quasi-optimal estimation can be performed in lower order norms (Sobolev spaces.) Examples are furnished of the application of Laplace and biharmonic equations for elliptical BVP.

Stepping from the standard FE approximation of elliptical BVP, one investigates a class of "nonstandard" FE methods, i.e. mixed methods. The latter are important since they can be utilized in providing excellent examples of how the general approximate results (Chapter 1) and interpolation theory (Chapter 2) can be applied in studying the various types of FE methods. Mixed element methods have important applications in treating incompressible fluids, solids, and contact problems in elasticity. These are subjects of other volumes in this series. Moreover, FE methods are looked upon as an approximation theory for linear variational BVP problems with constraints. The saddle point problems and existence theory are some of the ideas of constrained minimization. They and Lagrange multipliers play important roles in the mixed methods. Our attention is next directed toward the approximation and error estimates of constrained problems. Then, the important Bakuška-Brezzi conditions are expanded upon. Examples in incompressible elasticity and fluid flow are next considered in the  $C^0$  (conforming) approximations. The chapter concludes with a mixed method for fourth-order problems. This avoids the use of higher-order conforming FE.

The final chapter deals with another mixed method, i.e., hybrid methods. In previous chapters, the derivatives of the approximate solution must be continuous across interelement boundaries. Employing Lagrange multipliers, one can construct independent FE approximations on the interior of each element. Its various directional derivatives would be normal to the interelement boundaries. The author begins with some preliminaries, viz. partition of the domain into subdomains. The chapter continues with the hybrid variational principle. This sets the stage for the hybrid FE method. The chapter concludes with implementation of the derived hybrid FE accompanied by error estimates plus the stability of the hybrid methods.

As mentioned in the introduction, strict concentration is re-

quired in comprehending the mathematics underlying the FE methods. This book does accomplish the task that the authors set out to do. The reviewer feels that a glossary of the mathematical symbols would be helpful to the reader. Another benefit would be the addition of applicable examples detailed in a simple fashion, i.e. dynamics and heat transfer. Scant mention is made of the important isoparametric elements. This book should be of great assistance to the student or practitioner interested in comprehending the mathematical basis for FE.

**Structural Mechanics Software Series (Volume V).** by N. Perone and W. Pilkey, Ed., University Press of Virginia, Charlottesville, VA, 1985, 342 pages. Price: \$30.00

Have you ever found yourself in a predicament as to which finite element (FE) program to use in the analysis of complex structural shapes? Furthermore, you may not know if the program contains the necessary subroutines required in obtaining a solution. Most programs have a history of either mediocre or fair theoretical manuals. In recent years, the user's manuals have improved, although may still fail in furnishing impressive and desired features. In any FE program, the user's manual is important because it should leave a lasting impression with the user. For proper documentation, one must contain the following: (a) a nomenclature or glossary; (b) an index; (c) a recommended element list; (d) element summary table; (e) element displacement functions and other assumptions; (f) element benchmark problems; (g) convergence performance of elements; (h) evaluation of element via test results (single element, patch test, etc.); (i) user hints and avoidances; and (j) recommendations and strategies in meshing of suitable elements for different types of problems.

This volume is similar to the previous four volumes. As stated by the editors, "This volume contains current information and comparisons about programs and algorithms and procedures that should be of interest, as well as details about data bases, sources of data bases, sources of literature searches of various kinds, users groups . . .". Because of the continual updating of a number of well-known programs, plus the addition of specified FE programs applied to certain areas, this book and its predecessors are extremely helpful. They critique a number of programs and point out the good and bad features of the most popular ones. It takes off where the technical and users' manuals falter. The book consists of three main parts. Part I reviews and summarizes a number of available programs. Part II proceeds to review computational technology. The last part discusses resources for engineers. It points out to the reader the important sources of computational software.

Part I begins with a review of the capability of 38 computer programs used in heat transfer analysis. Beginning with the general purpose programs (NASTRAN, ANSYS, MARC, BERSAFE, ADINA, etc.), the authors continue with more specialized and smaller programs, such as TEMP and ASAHEAT. Analyses were made in tabular forms of the following contents: (a) date of last issue; (b) general information; (c) program capability and scope of analysis; (d) user interface and modeling capabilities; (e) solution methods; (f) limitations; and (g) programming languages and hardware/operating systems. This is very good chapter for those interested in heat transfer applications and FE methods.

The next paper reports on 18 computer programs for building analysis and design. Previously, most engineering design of buildings was performed by hand calculators and brute force. This is presently being supplanted by computers using structural engineering software. However, there is still a

need for well-documented, easy accessible, high-quality programs for applications to structural analysis and design of buildings. Most present computer programs are still confined to elastic analysis. However, plastic analysis programs are gradually coming to the fore because of its economy in the design of buildings. The author points out that most general-purpose programs are inefficient since they do not take the advantage of special characteristics of the more usual building structures and are expensive for building analysis. Consequently, the author focuses on 18 programs which are written by various organizations, engineering consultants, commercial software houses, and others. Among the more popular programs are STRESS, ICES/STRUDL, STRU-PAK program library, and others.

The third paper dwells on error estimates and solution of linear systems. This extends to properties of norms, determination of error estimates and the evaluation of computer programs on various selected test matrices. There are six FE programs, among them GIFTS-S, MARC, MSC/NASTRAN, and SAP IV. Two programs (SPARSPAK and CAL) are used in the solution of equations using matrix multiplication (sparse matrices.) The author spends a great deal of time discussing the LINPACK package of programs.

The fourth paper reports on eight general purpose FE programs: COSMIC/NASTRAN, MSC/NASTRAN, EASE 2, STARDYNE, ANSYS, ABAQUE, ADINA, and MARC. The author initially compares the documentation, the elements and composition of material libraries. Six of the eight codes are column solvers and the remaining two are wave front solutions. The paper concludes with time integration schemes for the eight programs. The author is extremely critical of these programs.

The last paper in Part I focuses on FE's use in biomechanics. FE has made a tremendous impact on the analysis of the mechanics of orthopedics, dental mechanics, impact injury, cardiovascular mechanics, biological flow, and soft tissue mechanics. There are 36 general and special programs that are critiqued. Another good chapter accompanied by an excellent set of references for each topic.

The initial paper in Part II consists of a concentrated and extensive evaluation of the STAGSC-1 computer program. The latter is for general purpose thin-shell structural analysis. It was primarily developed for nonlinear static and dynamic analysis. There are seven different analyses programs which include, in addition to the abovementioned C.P., bifurcation buckling analysis of a linear and nonlinear stress state. The assessment covers the general capabilities and its constituents, proper documentation of the program, program architecture functional evaluation, performance and analysis capability. The second paper is an extension of a previous review of recent numerical integration methods for stiff nonlinear systems. This includes the variable step difference algorithms and ordinary differential equations. Variants of both the Newmark and Runge-Kutta methods plus predictor-corrector methods are the next logical steps in the solution of nonlinear differential equations. The chapter concludes with the variable time step and variable order algorithm for nonlinear stiff equations, and the iteration schemes used in solving the differential equations. The last paper of Part II examines four procedures in reducing the number of degrees of freedom in solving dynamic problems. They are exact reduction, static condensation on Guyan's reduction, modified static condensation on modified Guyan's reduction and dynamic condensation method. The latter is an interesting theory pioneered by the author of the paper. The author makes no mention of Rubin's component mode representation and Hintz's component synthesis method.

Part III consists of four papers on data bases, software compliance, program dissemination centers and users groups plus computerized data bases. In summary, the authors and